points of the functional F.

By virtue of (10) the function f defines a function of the class C^2 on $T^n \times Z_0$, where Z_0 is a set of functions from Z with zero mean. According to (12), setting $z = \bar{z} + \zeta$, we have

$$f(\bar{z}, \zeta) = {}^1/_2\langle A\zeta, \zeta\rangle + g(\bar{z}, \zeta);\ \bar{z} \in T^n,\ \zeta \in Z_0$$

where $\langle A\zeta, \zeta\rangle$ is a non-degenerate quadratic form on $Z_0 = R^N$, and the partial derivatives of the function g are bounded for $\|\zeta\| \to \infty$. From this and the results in /3/ the assertion of the theorem follows.

The theorem can be extended to the case when the potential U is invariant relative to any crystallographic group G of transformations of the space R^n. In this case, Z^n must be replaced by G in the proof, and the Lyusternik-Shnirel'man category of the space R^n/G is the lower bound of the number of periodic solutions.

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ON TRANSONIC EXPANSIONS*

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The problem of finding the particular solutions of the linearized inhomogeneous transonic equations appearing in the transonic expansions, expressed explicitly in terms of the fundamental solution of the Karman-Fal'kovich (KF) equation, is discussed.

When the procedure of transonic expansion is used, e.g. in the thin-body theory /1/, the solutions of the equations of gas dynamics have the form of series in powers of a small parameter characterizing the measure of the deviation of the flow in question from homogeneous sonic, or a nearly sonic flow. To a first approximation, the non-linear KF equation has to be solved /2, 3/, and inhomogeneous linearized KF equations whose right-hand sides depend on the preceding terms are obtained for the higher-order approximations. It is convenient to have available an explicit expression for the particular solutions written in terms of the fundamental solution. Thus in /4/ two examples are given of determining the first correction in the theory of small perturbations for the axisymmetric flows of a compressible fluid when the correction is expressed in terms of the fundamental solution without taking into account its specific structure, and the uniqueness of such results is noted. The first-order correction to the solution of the KF equation was obtained in /5/.

In the case of plane parallel flow the KF equation reduces, in the hodograph plane, to the linear Tricomi equation, and the procedure of transonic expansion enables one, as was shown in /6, 7/, to determine particular solutions for any order of approximation. From this it follows that when transonic expansions are used, particular solutions of a general type can be obtained in the physical plane for the i-th approximation. The present communication does not demonstrate the procedure of passing from the hodograph expansions to expansions in the physical plane, but gives the following straightforward result: the first correction which is the same as that obtained in /5/, and the second correction. The fact that curvilinear integrals appear in the second correction but not in the first, is of interest.

In the case of an axisymmetric flow the first correction to the solution of the KF equation has the same form as in the plane parallel case. However, attempts, using the analogy with the plane-parallel case, to find the second correction in general form, have proved

**Prikl.Matem.Mekhan.*,51,4,688-690,1987

unsuccessfull. A change of variables is suggested, which considerably simplifies the differential equation used for its determination.

The plane-parallel and irrotational axisymmetric flows of an inviscid gas are described by the equation

$$-(\gamma+1)\varphi_x\varphi_{xx}+\varphi_{yy}+\omega\varphi_y/y=[(\gamma+1)\varphi_x^2/2+(\gamma-1)\varphi_y^2/2]\varphi_{xx}+ \\ 2(1+\varphi_x)\varphi_y\varphi_{xy}+[(\gamma+1)\varphi_y^2/2+(\gamma-1)(\varphi_x+\varphi_x^2/2)]\varphi_{yy}+ \\ \omega(\gamma-1)(2\varphi_x+\varphi_x^2+\varphi_y^2)\varphi_y/(2y) \tag{1}$$

where x, y are Cartesian or cylindrical coordinates, φ is the perturbation potential of the sonic flow, ω is a parameter equal to unity or zero respectively for plane and axisymmetric flow, and γ is the ratio of the specific heats of the gas.

In the near-sonic range of velocities Eq.(1) is replace by the KF equation

$$-(\gamma+1)\varphi_{0x}\varphi_{0xx}+\varphi_{0yy}+\omega\varphi_{0y}/y=0 \tag{2}$$

Let the solution φ_0 of Eq.(2) be known. We will construct the solution φ of Eq.(1) in the form of a series in powers of the small parameter ε with principal term containing the function φ_0

$$\varphi=\varepsilon^3\varphi_0(\bar{x},\bar{y})+\varepsilon^5\varphi_1(\bar{x},\bar{y})+\varepsilon^7\varphi_2(\bar{x},\bar{y})+\ldots \\ \bar{x}=x/\varepsilon,\quad \bar{y}=y \tag{3}$$

Henceforth we shall omit the bars above the symbols. The corrections $\varphi_1, \varphi_2, \ldots$ satisfy the linear inhomogeneous equations which are obtained by substituting (3) into (1)

$$K(\varphi_1)=(2\gamma-1)(\gamma+1)\varphi_{0x}^2\varphi_{0xx}/2+2\varphi_{0y}\varphi_{0xy} \tag{4}$$

$$K(\varphi_2)=(\gamma+1)\varphi_{1x}\varphi_{1xx}+(\gamma-1/2)(\gamma+1)(\varphi_{0x}^2\varphi_{1x})_x+2(\varphi_{0y}\varphi_{1y})_x+ \\ [(1/2)(\gamma-1)\varphi_{0y}^2+(\gamma^3-\gamma)\varphi_{0x}^3]\varphi_{0xx}+2\gamma\varphi_{0x}\varphi_{0y}\varphi_{0xy} \tag{5}$$

$$K(\Phi)\equiv-(\gamma+1)\varphi_{0x}\Phi_{xx}-(\gamma+1)\varphi_{0xx}\Phi_x+\Phi_{yy}+\omega\Phi_y/y$$

Let us try to find the particular solutions $\varphi_1, \varphi_2, \ldots$ of Eqs.(4) and (5) in general form, expressing them in terms of the function φ_0, its derivatives and integrals. Then formula (3) will represent the operator of the passage from the solution of the approximate KF Eq.(2) to the solution of the exact Eq.(1).

In particular integral of Eq.(4) has the form /5/

$$\varphi_1=Ay\varphi_{0x}\varphi_{0y}+B\varphi_0\varphi_{0x} \\ A=(2\gamma+5)/10,\ B=(-2\gamma+5)/10,\ \omega=0 \\ A=(2\gamma+5)/4,\ B=1,\ \omega=1 \tag{6}$$

Let us substitute the function φ_1 given by formula (6) into the right-hand side of (5). This will give us an inhomogeneous equation for φ_2 with the right-hand side consisting of 24 terms

$$K(\varphi_2)=A^2(\gamma+1)y^2\varphi_{0x}\varphi_{0y}\varphi_{0xx}\varphi_{0xxy}+\ldots \tag{7}$$

where repeated dots denote the remaining 23 monomials on the right-hand side.

Let us write the particular solution of Eq.(7) in the form

$$\varphi_2=[\varphi_1^2/(2\varphi_{0x})+(1/8)(\gamma+1)A^2y^2\varphi_{0x}^4]x+\varphi_2^*(x,y) \tag{8}$$

Substituting (8) into (7) we obtain an inhomogeneous equation for φ_2^* with a simpler right-hand side consisting of three terms

$$K(\varphi_2^*)=E_1\varphi_{0x}^3\varphi_{0xx}+E_2\varphi_{0y}^2\varphi_{0xx}+E_3\varphi_{0x}\varphi_{0y}\varphi_{0xy} \\ E_1=(\gamma+1)(8\gamma^2+5)/20,\ E_2=(\gamma+1)/2,\ E_3=2(\gamma+1);\ \omega=0 \\ E_1=(\gamma+1)(4\gamma^2+4\gamma+7)/8,\ E_2=(3\gamma+6)/2,\ E_3=4\gamma+7;\ \omega=1 \tag{9}$$

When $\omega=0$, the particular solution of (9) becomes

$$\varphi_2^*=Cx\left(\frac{\varphi_{0x}^3}{6}+\frac{\varphi_{0y}^2}{2(\gamma+1)}\right)+D(-\varphi_{0y}I_1+I_2) \\ C=-(24\gamma^2+70\gamma+85)/140,\ D=(-24\gamma^2+70\gamma+55)/140 \\ I_1=\int\left(\frac{\varphi_{0y}}{\gamma+1}dx+\frac{\varphi_{0x}^2}{2}dy\right) \\ I_2=\int\left[\left(\frac{\varphi_{0x}^3}{6}+\frac{\varphi_{0y}^2}{2(\gamma+1)}\right)dx+\frac{\varphi_{0x}^2\varphi_{0y}}{2}dy\right] \tag{10}$$

Here we use a curvilinear integral of the second kind independent of the path of integration between the points $(0, 0)$ and (x, y).

If φ_0 is a selfsimilar solution, then φ_i will also have a selfsimilar form and expansion (3) will be written in the form (n is the selfsimilarity index, $\zeta = xy^{-n}(\gamma+1)^{-1/3}$ is the selfsimilar variable)

$$\varphi = y^{3n-2} f_0(\zeta) + y^{5n-4} (\gamma+1)^{-1/3} f_1(\zeta) + y^{7n-6} (\gamma+1)^{-2/3} f_2(\zeta) + \ldots$$

Using (6), (8) and (10) we obtain the form f_1, f_2 (a prime denotes differentiation with respect to ζ)

$$f_1 = a_1 f_0 f_0' + a_2 \zeta f_0'^2 \quad (\omega = 0,\ \omega = 1)$$
$$a_1 = A\,(3n-2) + B, \quad a_2 = -nA$$

$$f_2 = b_1 \zeta^3 f_0'^2 + b_2 \zeta^2 f_0 f_0' + b_3 \zeta f_0^2 + b_4 \zeta f_0'^3 + b_5 f_0 f_0'^2 + b_6 \zeta f_0 f_0' f_0'' + b_7 f_0'^3 f_0'' + b_8 f_0^2 f_0'' \quad (\omega = 0)$$
$$b_1 = n^2 D/2 + (-10n + 9) n^3 C/(2H)$$
$$b_2 = (-6n + 4) b_1/n, \quad b_3 = (3n-2)^2 b_1/n^2$$
$$b_4 = 11n\,(n-1) A^2/2 - nAB + D/6 + (29n - 24) nC/(6H)$$
$$b_5 = (5n-4) AB + 5\,(3n-2)(-n+1) A^2/2 + B^2 + (3n-2)(-3n+3) C/(2H)$$
$$b_6 = 2 a_1 a_2, \quad b_7 = 2A^2, \quad b_8 = a_1^2/2$$
$$H = (7n-6)(4n-3)$$

The method of expanding the solution of (1) in a series in selfsimilar components is widely used, beginning with /8/, but the form of the corrections f_1, f_2 was found only for some particular values of n.

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ON MATHEMATICAL MODELS OF MAGNETIC FLUIDS*

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A system of equations describing models of magnetic fluids (MF) with internal angular momentum in a magnetic field is studied. Linearized equations and their solutions in the form of spin waves and magnetosonic waves are given. The high-frequency magnetic susceptibility tensor of the fluid is calculated and the frequencies of homogeneous magnetic resonance are determined. The connection between the spin and acoustic waves in MF is governed by the presence, in the internal energy of the fluid, of terms with vorticity vector and deformation rate tensor (determining, in particular, the hydromagnetic energy). Various existing models used to describe ferromagnetic fluids (FMF) are discussed. Relaxation models of MF are studied and used to obtain the solutions of problems of plane Couette flow and cylindrical Poiseuille flow. A new expression for the effective viscosity of the MF is obtained.

Several different models of MF are known. The simplest model /1/ describes paramagnetic fluids and certain types of the FMF in quasistationary magnetic fields quite well. However, in a number of important cases the above model cannot be used (e.g. at high frequencies of the magnetic field and for FMF at high volume concentrations of ferromagnetic particles with a

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